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STRESS CONCENTRATION AND FATIGUE LIFE PREDICTION FOR DISKS CENTRIFUGAL SEPARATORS

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Abstract

The paper presents the continuation of a previous study (ANGHEL, IATAN, PASAT, 1998) concerning the elastic analysis of disk centrifugal separators. The goals are the state of stress analysis and stress concentration determination in the main critical junctions of the separator's bowl, subjected to loads corresponding to the main technological conditions. Taking into account the cyclic nature of technological loads (centrifugal forces $r \cdot \omega^2$) standard procedures for estimating the fatigue and residual strength of the entire bowl are discussed for prediction of the lifetime. The numerical results presented were obtained for a real small separator's bowl. Two numerical analyses were developed to identify the critical junctions of the bowl due to the main loads: central axial load (F_a), centrifugal force ($r \cdot \omega^2$) and internal pressure (p). One analysis is based on the extension of the classical thin shell theory and the flexibility matrix method (ANGHEL, IATAN, PASAT, 1998) and the second on the finite element method (FEM), using a professional package, COSMOS/M Designer II. The study reveals a reasonable accuracy of the analytical and numerical results, an accurate positioning of the critical junctions and a great number of lifetime service cycles. On the other hand, the study may be a suitable method for preliminary design analysis and load-carrying capacity prediction of such structures.

Keywords: disk centrifugal separators, state of stress, critical junction, numerical analysis, flexibility matrix method, finite element method, fatigue life prediction, load-carrying capacity.

1. Introduction

Small disk centrifugal separators, having a flow capacity $Q_{\max} = 400 - 3000$ l/h with discontinuous or semi-continuous function are usually utilised in many fields of processing, such as food industry or drug industry. The study presents numerical results concerning only disk centrifugal separators with small flow capacity used for mechanical clarification and separation of milk (*Fig. 1a*), or in the separation of heterogeneous mixtures of liquid-oil phases. Without constructive modification (ANGHEL, 1996) these disk centrifugal separators may run at variable rotation speeds only with an electronic frequency converter. Thus, from the functional viewpoint, separations with these disk centrifugal separators consist of two steady-state stages, or separation phases:

- one stage with constant rotation speed n_1 used only for the mechanical clarification of milk;
- the second stage with constant rotation speed $n_2 > n_1$ used for complete separation.

The technological lifetime for these separators is roughly estimated at 16–20 years – due to the high demand of safety and performance. Due to the cyclic variation of rotational speeds these separators' structures are subjected to variable cyclic loading and therefore the fatigue behaviour occurs. The fatigue of the bowl's components is induced by the variation of the cyclic rotational speeds. In these conditions the classical calculation related to the fatigue strength, σ_R, becomes inconclusive, so that it will be replaced by the calculation of a limited service lifetime. The quantification of the zones with maximum stresses and displacements was distinctly analyzed using analytical and numerical (finite element FEM) methods, highlighting the cyclic nature of the loads and the fatigue behaviour of the separator's bowl. The lifetime until fractures N_r, under constant amplitude σ_i cycling, may be reasonably evaluated by the Coffin – Manson equation (JINESCU, 1984; RENERT, 1982). Furthermore, based on the general Palmgren – Miner cumulative damage theory under variable loads, the general prediction of the limited service lifetime was applied. Finally, based on solutions available in the literature (SOKOLOV, 1976; ANGHEL, 1998) and numerical analyses (finite element method – FEM) the critical rotational speed and the load-carrying capacity of the bowl were quantified.

2. Theoretical Approach

2.1. Application of the Analytical Method

The analytical method of linear elastic solution, suitable for this analysis has been descibed in detail in some previous studies (ANGHEL, IATAN, PASAT, 1998; ANGHEL, 1997). It has been stated (ANGHEL, IATAN, PASAT, 1998) that the structural elements of the bowl (*Fig. 1a, b*) are typically 'short or intermediate' and by the point of simplex order h/R they are called 'thin or moderate' because $h/R \leq 0.1 \ldots 0.33$. We can remember the previously established condition for the outline effect zone:

- for cylindrical shell zone $L_C \cong 2.7 \times (Rm \times h)^{0.5}$;
- for truncated conical shell zone

$$L_K \cong 3.635 \times (\cos\alpha/h \times \sin 2\alpha)^{0.5} \times (R_{\max}^{0.5} - R_{\min}^{0.5}). \qquad (1)$$

The main external loading conditions for the separator's bowl are: central axial load F_a induced from the nut on mounting, centrifugal force of the bowl $R \cdot \omega^2$ which rotates at a high speed ω, and the internal hydrodynamic pressure of the liquid p. This internal hydrodynamic pressure of the liquor, which rotates together with the bowl at a high speed ω, may be established from solutions available in the literature (ANGHEL, 1998; SOKOLOV, 1976). Thus for a separator's bowl with a continuous flow, in accordance with the condition that the internal hydrodynamic pressure of the liquor may be obtained from the centrifugal force of the fluid related to the unit area of the bowl, we can write:

$$p = 0.5 \cdot \rho_s \cdot \omega_T^2 (r_T^2 - R_0^2), \tag{2}$$

where ω_T – the rotational speed of the bowl, ρ_S – density of the liquor, R_0 – the radius of liquor surface inside of bowl and r_T – the customary radius of internal bowl. The customary radius of liquor surface inside of bowl may be obtained from the function of empty rate:

$$\psi = 1 - (R_0/r_T)^2. \tag{3}$$

Usually for disk centrifugal separators, typical values of the empty rate $\psi = 0.9 \ldots 1$ are available in the literature (SOKOLOV, 1976; GUSAKOV, RUTEPOV, 1975). Numerical integration of previous expressions (2)–(3) in lengthways of the bowl leads to concrete values for the internal hydrodynamic pressure of any point.

2.2. Finite Element Method (FEM)

Based on the geometrical and loading conditions of axial symmetry of the separator's bowl (*Fig. 1a*) only a quarter of the axial section of the bowl may be analysed. *Figs 2–10* present the pattern and the boundary conditions for the finite element analysis (FEM) – simple supports were considered in axial direction. For the prestressed stage with central load F_a the following conditions are considered:

- for the outside part of the bowl FEM analysis was carried out using solid parabolic triangular finite elements of size 3.913 mm with 9856 elements and 19772 nodes;
- for the inner bottom of the bowl FEM analysis was carried out using solid parabolic triangular finite elements of size 4.556 mm with 7231 elements and 12778 nodes.

Complex loading conditions with central axial load F_a, angular velocity ω and internal hydrodynamic pressure of the liquid p were considered under the following conditions:

- for the outside part of the bowl, FEM analysis was carried out using sectional axisymmetric parabolic-quad finite elements of size 1.005 mm with 668 elements on the axial section and 2457 nodes;
- for the inner bottom of the bowl, FEM analysis was carried out using sectional axisymmetric parabolic-quad finite elements of size 1.657 mm with 760 elements on the axial section and 2605 nodes;

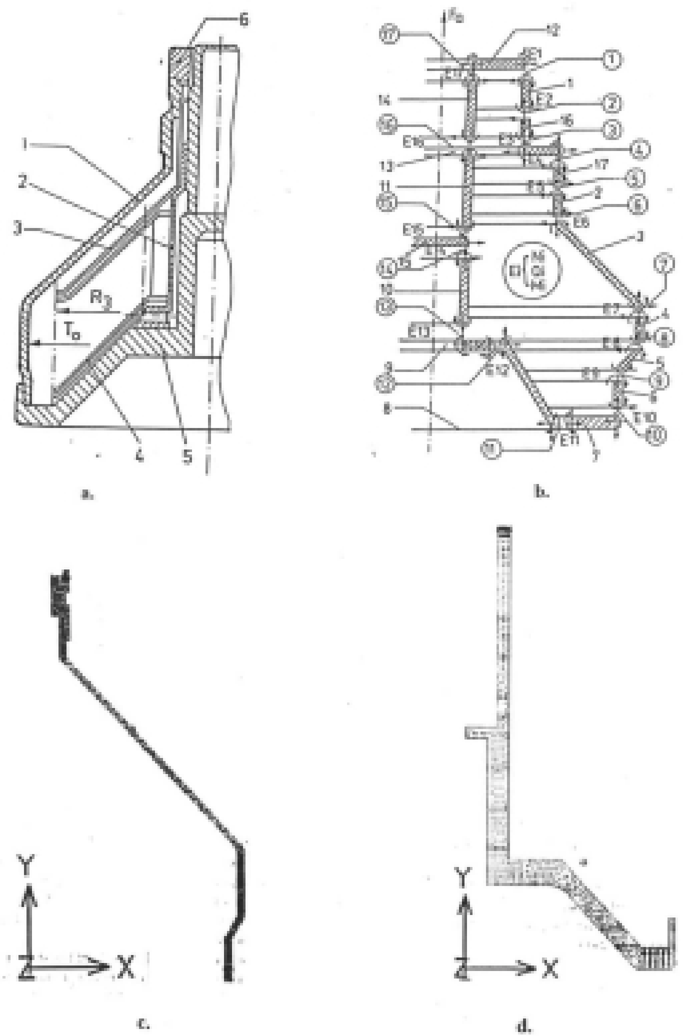

Fig. 1. Constructive, analytical and numerical design for separator disks bowl. a.) Cross-sectional half view of the bowl: 1-outside upper part of the bowl 'CS', 2-guiding disks mark, 3,4-tronconical separator disks, 5-inner lower part of the bowl 'CI', 6-central nut for assembling; b.) System of discrete elements of a bowl; c) FEM model mesh and boundary conditions of the outside upper part of the bowl 'CS'; d.) FEM model mesh and boundary conditions of the inner lower part of the bowl 'CI'

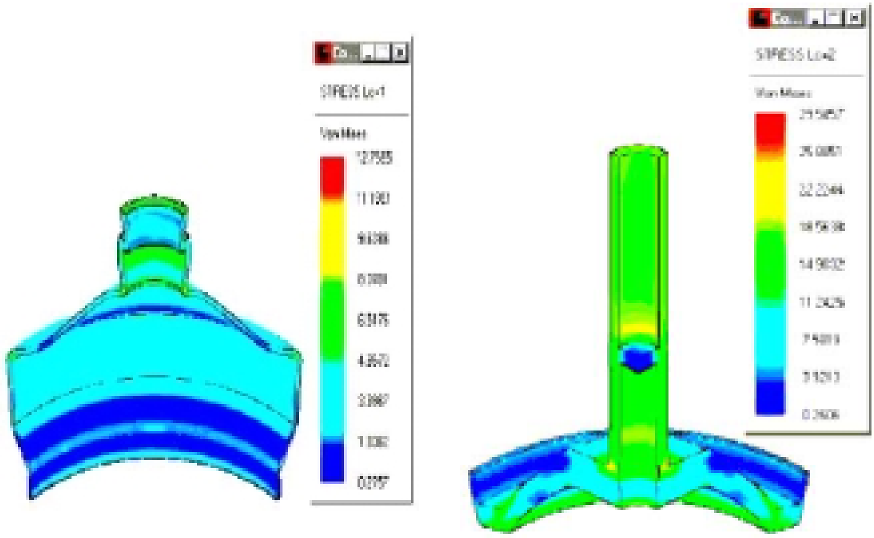

Fig. 2. Equivalent stress and deformed shape – FEM values. a.) Case 1 (*Table 1*) for the outside upper part of the bowl 'CS'; b.) Case 2 (*Table 1*) for the inner lower part of the bowl 'CI'

2.3. Stress Concentration

Due to structural discontinuities between the bowl's component elements – changes of the geometrical profile and variations of the elements' thickness (*Fig. 1a,b*) – the junctions are classified as critical areas (CIOCLOV, 1983; JINESCU, 1984) with possible strong stress concentration. Degradation effects, a decrease of mechanical strength – induced at these critical areas – are extremely dangerous especially when the external loads are variable and they also have a major effect on the corrosion resistance. We must mention that the single cyclic variable loads are considered as centrifugal force ($r \cdot \omega^2$), due to the angular velocity ω, corresponding to the steady-state stage ω_1 and ω_2.

Thinking of an elastoplastic stage, based on some general statements (JINESCU, 1984; RENERT, 1982), the stress concentration factor can be satisfactorily approximated using Neuber's formula:

$$\alpha^2 = \alpha_\sigma \alpha_\varepsilon, \tag{4}$$

where α is the general stress concentration factor considering a linear elastic behaviour of the material, α_σ is the pure stress concentration factor and α_ε the pure strain concentration factor.

If the values of stress are in the elastic domain, where ($\sigma_{\max}$ and $\sigma^*_{\max}$) $< \sigma_{0.2}$, the general stress concentration factor α becomes the elastic stress concentration

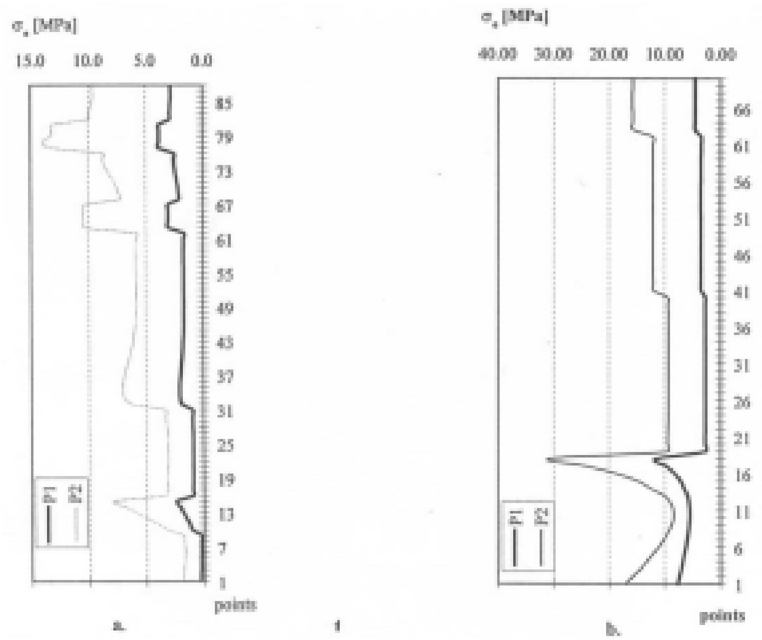

Fig. 3. Equivalent stresses – analytical modelled values. a.) Case 1,2 (*Table 1*) for the outside upper part of the bowl 'CS'; b.) Case 1,2 (*Table 1*) for the inner lower part of the bowl 'CI'

factor when $\alpha \cong \alpha_\sigma$. According to some usual design standards and other statements (PAVEL, 1998; CIOCLOV, 1983), this elastic stress concentration factor may be considered for a rapid assessment of the stress concentration factor, by a simplified form:

$$\alpha \cong \alpha_\sigma = \frac{\sigma^*_{\max}}{\sigma_{in}}, \tag{5}$$

where $\sigma^*_{\max}$ is the maximum effective value of the normal equivalent stress in the critical area, estimated by the Coulomb – Tresca maximum tangential stress criterion and σ_{in} is the maximum value for the nominal stress (hoop membrane stress) in the same area. In accordance with previously mentioned works, the general stress concentration factor may be expressed as:

$$\alpha \cong \alpha_{2\sigma} = \frac{\sigma^*_{2\max}}{\sigma_{2\max}}, \tag{6}$$

where $\sigma^*_{2\max}$ represents the maximum effective value of the normal hoop stress in the critical area with stress concentration, and $\sigma_{2\max}$ represents the value of the normal hoop stress in the area without stress concentration. Because, roughly speaking, all the structural elements of the bowl are typically 'short or intermediate' (ANGHEL, IATAN, PASAT, 1998) the outline effect area spreads along the whole length, for the structures of this separator's bowl and the membrane stress stage is insignificant. In accordance with the preceding conditions, based on the numerical results (*Figs. 4–9*) and other works (CIOCLOV, 1983; SENSMEIER, TIBBALS, 1999) as long as the values of stress are in the elastic domain, where $(\sigma_{\max}, \sigma^*_{\max}, \sigma^*_{2\max}) < \sigma_{0.2}$, and the value of the normal hoop stress σ_2 is positive and relatively close to the normal equivalent stress, the stress concentration is one elastic stress concentration and we may consider a simplified form:

$$\alpha \cong \alpha_{m\sigma} = \frac{\sigma^*_{\max}}{\sigma_{\max}}, \tag{7}$$

where $\sigma^*_{\max}$ is the maximum effective value of the normal equivalent stress in the critical area and $\sigma_{\max}$ is the maximum effective value of the normal equivalent stress in the area without stress concentration. As a customary position point for values of stresses $\sigma_{\max}$ in the area without stress concentration, we shall consider any structural point 'l_x' for which:

- for cylindrical shell zone $l_x \geq L_C \cong 2.7 \times (Rm \times h)^{0.5}$;
- for truncated conical shell zone

$$l_x \geq L_K \cong 3.635 \times (\cos\alpha/h \times \sin 2\alpha)^{0.5} \times (R^{0.5}_{\max} - R^{0.5}_{\min}). \tag{8}$$

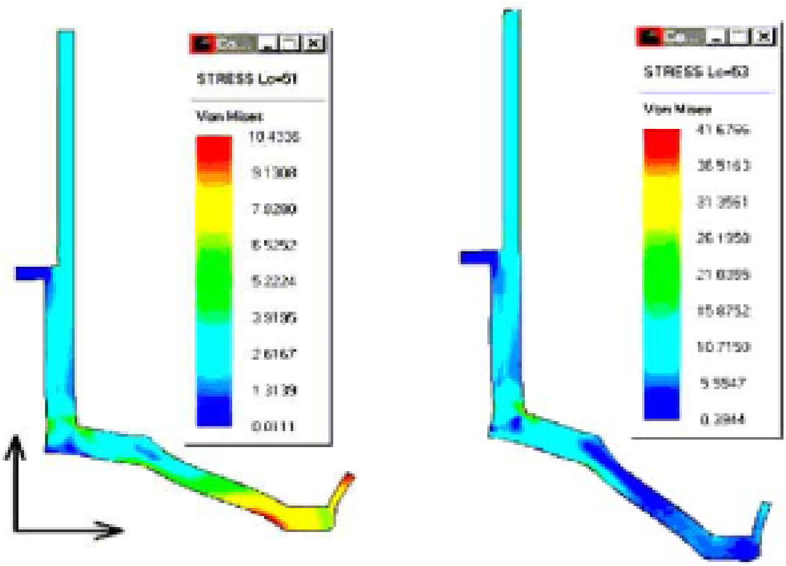

Fig. 4. Equivalent stress and deformed shape for the inner lower part of the bowl – FEM values. a.) Case 51 (*Table 1*); b.) Case 53 (*Table 1*)

The customary position point for any structural element of the bowl 'l_x' is defined in *Table 1* (ANGHEL, IATAN, PASAT, 1998).

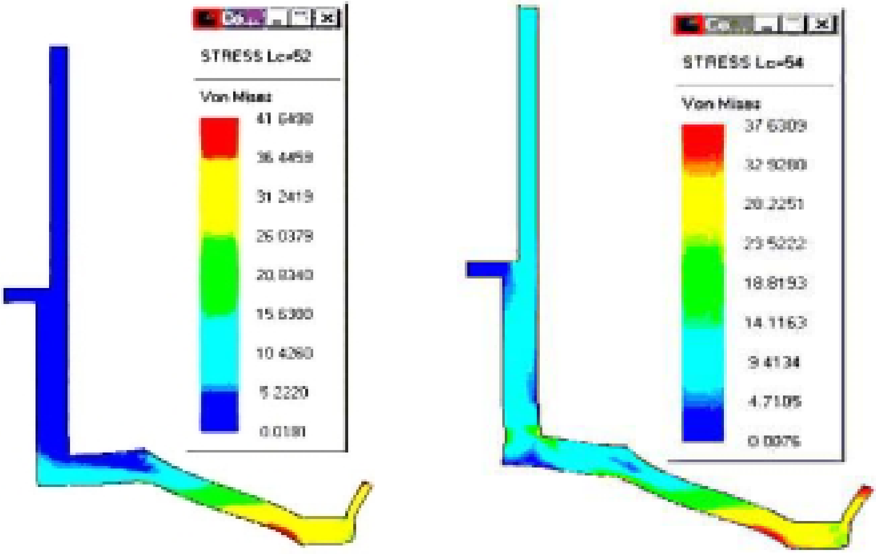

Fig. 5. Equivalent stress and deformed shape for the inner lower part of the bowl – FEM values. a.) Case 52 (*Table 1*); b.) Case 53 (*Table 1*)

Table 1. Pre-set technological and modelling parameters for fatigue life prediction

Technological lifetime [years]	Cyclic separation time [hour]	Daily cycles	Technological time [month/year]	Average rate [days/month]	Cyclic lifetime N_u
16	2	7	9.4	30	$3.158 \cdot 10^4$
20	2	7	9.4	30	$3.948 \cdot 10^4$
Load case/Mounting and technological parameters					
Case 1	Prestressed stage with central load $F_{a1} = 1480$ N				
Case 2	Prestressed stage with central load $F_{a2} = 5120$ N				
Case 51	Prestressed stage $F_{a1} = 1480$ N and rotation with $\omega_1 = 470$ s^{-1}				
Case 52	Prestressed stage $F_{a1} = 1480$ N and rotation with $\omega_2 = 890$ s^{-1}				
Case 53	Prestressed stage $F_{a2} = 5120$ N and rotation with $\omega_1 = 470$ s^{-1}				
Case 54	Prestressed stage $F_{a2} = 5120$ N and rotation with $\omega_2 = 890$ s^{-1}				

Note F_{a1} – minimum central axial load induced from the nut on mounting, F_{a2} – maximum central axial load induced from the nut on mounting; ω_1 – the rotational speed of the bowl for the mechanical clarification when $n_1 = 4500$ rpm; ω_2 – the rotational speed of the bowl for complete separation when $n_2 = 8500$ rpm

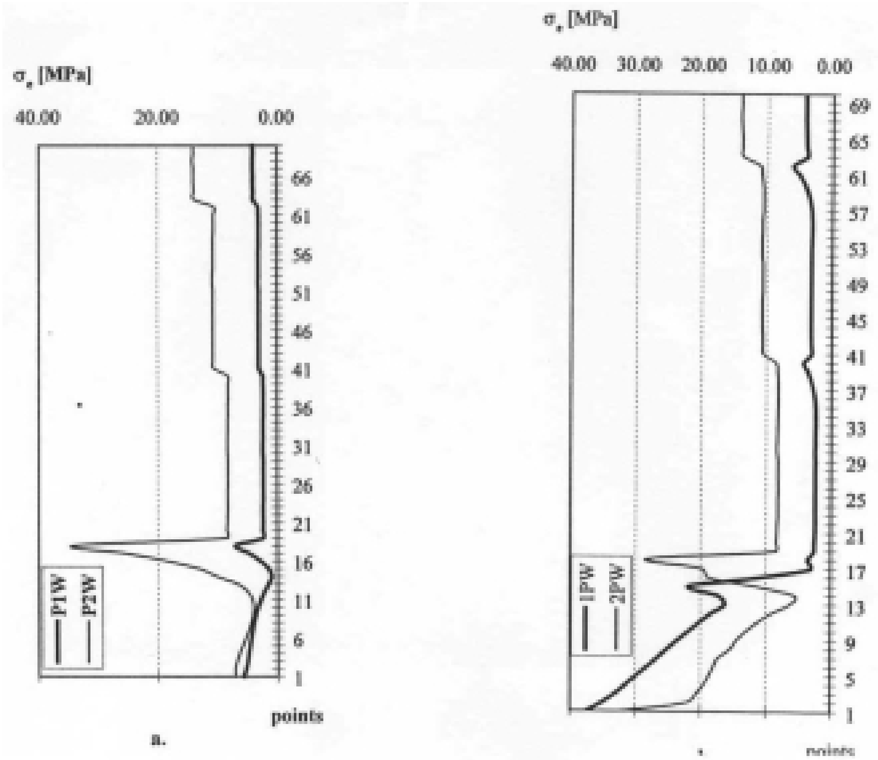

Fig. 6. Equivalent stress and deformed shape for the inner lower part of the bowl – analytical modelled values. a.) Case 51, 53 (*Table 1*); b.) Case 52, 54 (*Table 1*)

2.4. *Fatigue Analysis*

Stemming from the technological functional process, separations with these disk centrifugal separators have cyclic steady-state phases, due to the angular velocity ω_1 and ω_2. Thus the separator's structure is subjected to variable cyclic stresses and the fatigue behaviour appears. The aim of our analysis is to illustrate a general fatigue process for the entire structure of the bowl under cyclic loads in general terms, without illustrating the fatigue by typical approaches of fracture mechanics and crack propagation. This procedure could be used for a rapid assessment of the strength and endurance of any existing separator's bowl in order to establish the performance and the safe lifetime or for designing a new one. Based on the general Palmgren – Miner cumulative damage theory (JINESCU, 1982; HASIN, 1980) under

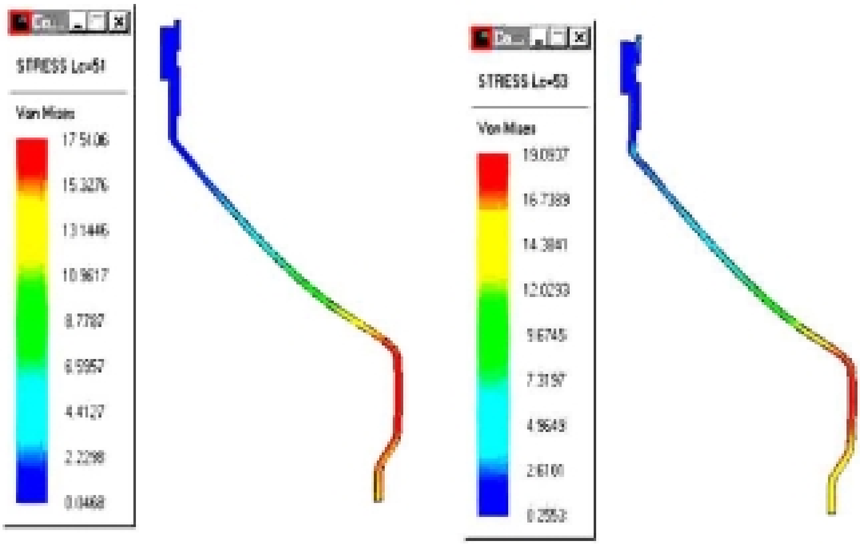

Fig. 7. Equivalent stress and deformed shape for the outside upper part of the bowl – FEM values. a.) Case 51 (*Table 1*); b.) Case 53 (*Table 1*)

variable loads, the general predicting equation is:

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = a = \text{const.}, \tag{9}$$

where n_i is the actual number of cycles at constant amplitude σ_i, N_i the lifetime for constant amplitude σ_i cycling, k is the number of constant amplitude cycles, and a a sub-unit constant value which depends among other things on the number of steps in range loading. The lifetime until fracture N_r, under constant amplitude σ_i cycling, may be reasonably evaluated by the Coffin – Manson equation (JINESCU, 1984; RENERT, 1982). If the values of the stress are in the elastic domain, where ($\sigma_{i\,\max}$ and $\sigma^*_{i\,\max}$) $< \sigma_{0.2}$, the Coffin – Manson formula may be applied in a simplified form:

$$\varepsilon_e \cong \frac{3.5 \cdot \sigma_r(T)}{E} N_r^{-0.12}; \tag{10}$$

where $\sigma_r(T)$ is the breaking strength of the material and E the modulus of elasticity at the working temperature T. In the previous context, usually for technological equipment with similar structures for use in process industries, for design purposes for temperatures T lower than the creep temperature T_{fl} we can consider $a = 0.80\ldots\ldots 0.5$ and the admissible designing lifetime:

$$N_a \cong 0.3 N_r. \tag{11}$$

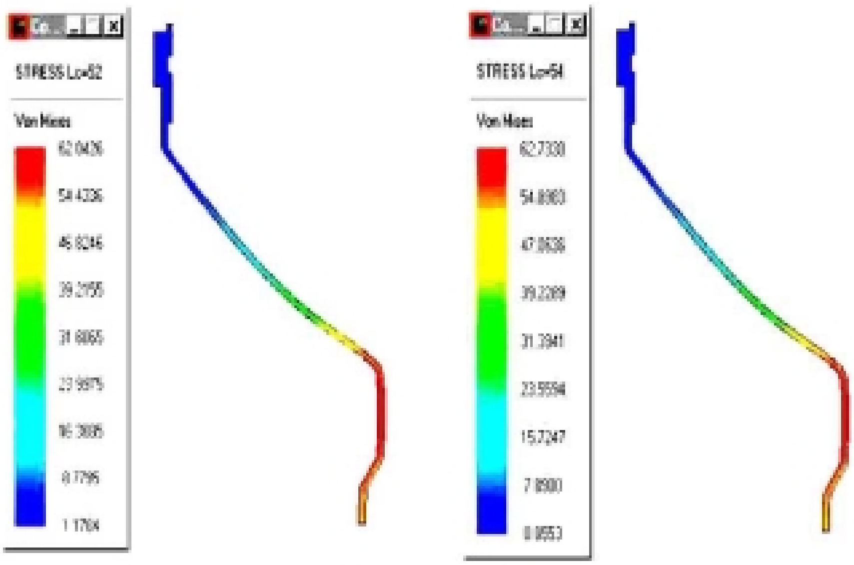

Fig. 8. Equivalent stress and deformed shape for the outside upper part of the bowl – FEM values. a.) Case 52 (*Table 1*); b.) Case 54 (*Table 1*)

3. Numerical Approach

The numerical analysis was carried out for a relatively small centrifugal separator, type TSL-400 or SECEL-4 Tehnofrig S.A, with flow capacity $Q_n = 400$ l/h at various rotation speeds $n = 4500 \ldots 8500$ rpm ($n_1 = 4500$ rpm for mechanical clarification and $n_2 = 8500$ rpm for complete separation). For an average function of empty rate $\psi = 0.9 \cong$ constant the pre-set working analysis parameters are listed in *Table 1*. The bowl of the separator is made of usual steel OL 37.2k, STAS 500/2-80 having a good capacity of deformation. For simplicity, according to well-known considerations, the results are presented in Mises equivalent stresses and only occasionally in normal hoop stress.

3.1. The State Stress Distribution

The analytical and numerical (FEM) results for the state of stress, corresponding to various loading cases (*Table 1*), are presented graphically in (*Figs. 2–10*). In general an acceptable correlation was established between the analytical and numerical results, but according to our and other researchers' (CIOCLOV 1983; MELERSKI, 1991, 1992) expectations, the maximum analytical nominal or equivalent stresses, in the main junction are more than 30%–50% higher than the FEM results. The analytical and numerical (FEM) results, presented graphically (*Figs. 4–9*), are reasonable and below the allowable stress of the material (*Table 2*). They reveal the following main trends for the state of stress:

- for both main parts ofthe separator bowl, marked 'CS' and 'CI', the stress generally increases with increasing rotational speed ω and central axial load F_a induced on mounting;
- a high increase in the central axial load F_a – under the same conditions of rotational speed ω – has a favourable result in the decrease of the state of stress for the main 'CI' part of the bowl of the separator;
- large areas with stress concentration are noticed on the main 'CI' part of the bowl between the sections '1e–1y' and '1e–1z' (*Fig. 10b*), simultaneously with a smooth evolution of the stress concentration areas on the main 'CS' part of the bowl between the sections '1e–1z' and '3e–3z' (*Fig. 10a*);

Table 2. Maximum stresses and geometric positions (in accordance with *Figs. 4,5 –7,8,10*)

Load case	$\sigma^*_{e\,\max}$ [MPa]	$\sigma_{e\,\max}$ [MPa]	$\sigma^*_{2\,\max}$ [MPa]	$\sigma_{2\,\max}$ [MPa]	σ_{ni} [MPa]
For the outside upper part of the bowl 'CS'					
Case 51	17.52/'1e'	17.30	16.87/'1z'	16.60	1.75
Case 52	62.04/'2e'	61	59.94/'1z'/	59.20	9.34
Case 53	19.09/'3e'	16.53	15.59/'3z'/	15.59	1.75
Case 54	67.73/'1e'	61.90	59.60/'1z'/	59.60	9.34
For the inner lower part of the bowl 'CS'					
Case 51	10.43/'1e'	8.40	10.61/'1z'	8.70	1.35
Case 52	41.65/'1e'	34.60	44.66/'1z'	33.63	4.82
Case 53	41.67/'1y'	11.75	36.64/'1y'	15.37	1.35
Case 54	37.63/'1e'	31.40	38.13/'1z'	30.50	4.82

3.2. *Stress Concentration*

The analysis based on the linear-elastic classical flexibility matrix method leads to results which signify normal average stresses in the section area and these values do not really reflect the stress concentration. For this reason only the values of stresses obtained by the finite element method (FEM) were considered for the stress concentration analysis (*Tables 2, 4* and *Figs. 4–8*). For the stress concentration factor given by expression (5) we have considered, like (CIOCLOV, 1983; PAVEL, 1998), an equivalent conventional load produced by the internal pressure p, (*Table 1*). The stress concentration factors given by this expression (5) for α_σ, having values between 6.420 . . . 27.140, represent significant values for the stress concentration. However, the stress concentration factors given by expression (5) are not very informative because for this separator's bowl the membrane stress stage is insignificant or conventional. The stress concentration factors given by expressions $\alpha_{2\sigma}$ (6) and $\alpha_{m\sigma}$ (7) are of real interest. These values of the stress are realistic and below the allowable stress of the material. Because relatively small values were established for the main 'CS' part of the bowl (between 1.013 ... 1.155), and for the main 'CI' part of the bowl (1.219 ... 3.577), it is thought that a normal level of stress concentration is available for the bowl of this separator. At the same time the stress concentration state is in good agreement with some restrictions (SENSMEIER, 1999), $\alpha_{2\sigma}, \alpha_{m\sigma} \leq 7$, referring to the same stress concentration factors $\alpha_{2\sigma}$, $\alpha_{m\sigma}$. On the other hand, these areas with the highest stress concentration factors are localised in the same areas in which other experimental works (SOKOLOV, 1976; GUSAKOV, RUTEPOV, 1975) reveal the beginning of the cracking of the bowl.

Table 3. Geometric positions of maximum stresses (*Fig. 10*)

Location/ Section [mm]	Parts of the bowl							
	Outside upper part 'CS'					Inner lower part 'CI'		
	1e	1z	2e	3e	3z	1e	1z	1y
x	96	99	96	95.30	97	66.60	72.80	18.35
y	39.52	39	38.52	53.20	54.67	4.40	8.70	43.88

Table 4. Elastic stress concentration factors

Factor α/ Load case	α_σ	$\alpha_{2\sigma}$	$\alpha_{m\sigma}$
For the outside upper part of the bowl 'CS'			
Case 51	9.640	1.016	1.013
Case 52	6.420	1.013	1.017
Case 53	10.14	1.139	1.155
Case 54	6.470	1.014	1.094
For the inner lower part of the bowl 'CI'			
Case 51	7.850	1.219	1.242
Case 52	9.260	1.328	1.204
Case 53	27.14	2.384	3.577
Case 54	7.90	1.250	1.198

3.3. The Limited Service Lifetime

In close correlation with the maximum state of the stress (*Table 2* and *Figs. 4–8*), our procedure for the fatigue life prediction was considered for idealised multistage loading steps, for a stepwise constant amplitude cycling $\sigma = \sigma_{\max}$. Because the lifetime of the apparatus is between 16–20 years, – due to the high demand for safety and performance – the use of the separator's bowl is reduced only to a relatively small number of cycles $N < 4 \cdot 10^4$ (*Table 1*), so the classical approach related to the fatigue strength, σ_R, becomes inconclusive. It will be substituted by the calculation of a limited service lifetime based on the previous expressions (9)–(11). Even for the most stressed areas (*Tables 5–6*) this type of separator bowl allows a significantly greater number of lifetime cycles than the whole bowl's technological lifetime, $N_a = 1.15 \cdot 10^9 \gg N = 3.95 \cdot 10^4$. For the two considered technological steady-state stages, maximum values of the stresses occur in critical areas and they are much less than the technical yield stress $\sigma_{0.2} = 230$ MPa. Thus under normal technological conditions probably a process like the 'creeping effect' may cause an increase in the fatigue endurance limit.

Table 5. Theoretical maximum lifetime (cycles) for the general structure of the bowl

Location/ Load case	For the outside upper part of the bowl 'CS'		For the inner lower part of the bowl 'CI'	
	$N_{ar}(\sigma^*_{2\max})$	$N_{ar}(\sigma^*_{e\max})$	$N_{ar}(\sigma^*_{2\max})$	$N_{ar}(\sigma^*_{e\max})$
Case 51	$1.20 \cdot 10^{15}$	$8.82 \cdot 10^{14}$	$5.73 \cdot 10^{16}$	$6.51 \cdot 10^{16}$
Case 52	$3.09 \cdot 10^{10}$	$2.31 \cdot 10^{9}$	$3.54 \cdot 10^{11}$	$6.39 \cdot 10^{11}$
Case 53	$7.83 \cdot 10^{14}$	$4.26 \cdot 10^{14}$	$1.88 \cdot 10^{12}$	$6.42 \cdot 10^{11}$
Case 54	$2.88 \cdot 10^{10}$	$1.12 \cdot 10^{10}$	$1.34 \cdot 10^{12}$	$1.49 \cdot 10^{11}$

Table 6. Theoretical admissible lifetime for the general structure of the bowl

Location/ Load case	For the outside upper part of the bowl 'CS'		For the inner lower part of the bowl 'CI'	
	$N_a(\sigma^*_{2\max})$	$N_a(\sigma^*_{e\max})$	$N_a(\sigma^*_{2\max})$	$N_a(\sigma^*_{e\max})$
Case 51, 52	$1.54 \cdot 10^{10}$	$1.15 \cdot 10^{9}$	$1.77 \cdot 10^{11}$	$3.20 \cdot 10^{11}$
Case 53, 54	$1.44 \cdot 10^{10}$	$5.59 \cdot 10^{9}$	$3.91 \cdot 10^{11}$	$6.04 \cdot 10^{10}$

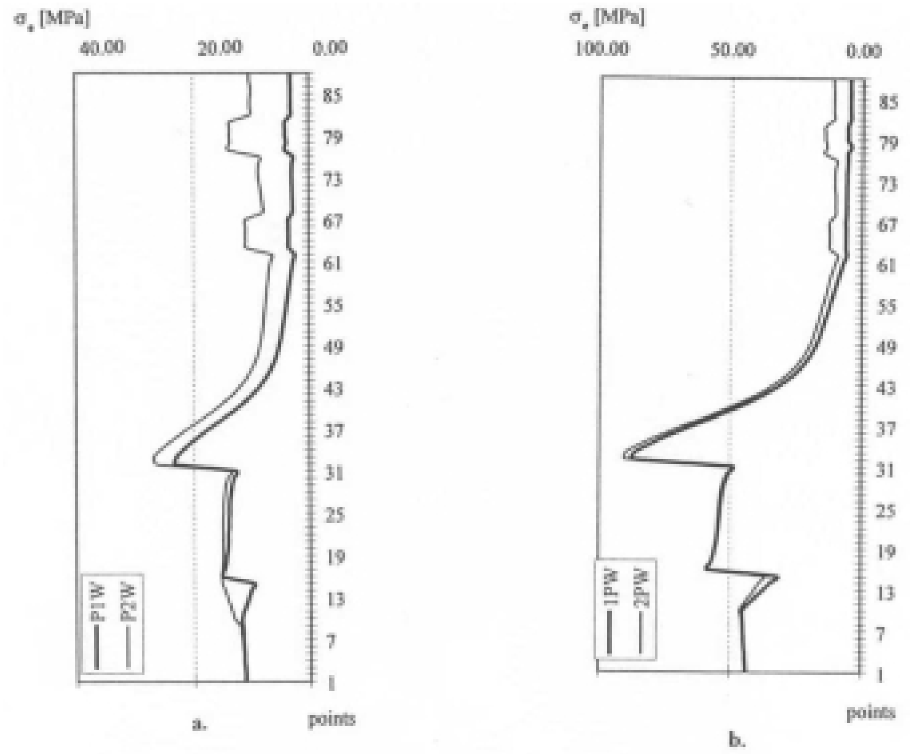

Fig. 9. Equivalent stress for the outside upper part of the bowl – analytical modelled values. a.) Case 51, 53 (*Table 1*); b.) Case 52, 54 (*Table 1*)

3.4. Load-Carrying Capacity of the Bowl

Based on solutions available in the literature (SOKOLOV, 1976; ANGHEL, 1998) a rough estimation of the critical rotational speed – only for each independent structural component of the bowl – can be made by using the general predicting equations:

$$n_{cr} = 9.55 \cdot R^{-1} \cdot \left[\sigma_a^*(0.5 \cdot \rho_l \cdot R_0 \cdot \psi \cdot h^{-1} + \rho_m)^{-1}\right]^{0.5}, \qquad (12)$$

for cylindrical structural elements and

$$n_{cr} = 9.55 \cdot \left[\sigma_a^*(0.5 \cdot \rho_l \cdot (R^2 - R_0^2) \cdot R \cdot h^{-1} \cdot \cos^{-1}\alpha + \rho_m \cdot R^2)^{-1}\right]^{0.5} \qquad (13)$$

for conical structural elements. According to the considered values for the conventional allowable stress limit as $\sigma_a^* = \sigma_a = 160$ MPa, or for the technical yield stress $\sigma_a^* = \sigma_{0.2} = 230$ MPa, or for the breaking strength $\sigma_a^* = \sigma_r = 370$ MPa,

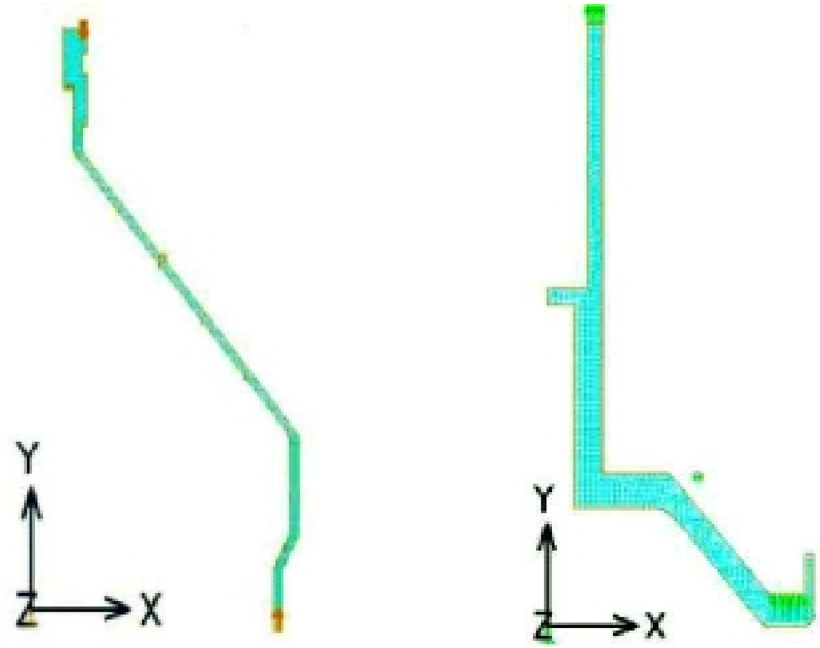

Fig. 10. The location of the areas of the critical junction for equivalent Mises stresses. a.) For the outside upper part of the bowl 'CS'; b.) For the inner lower part of the bowl 'CI'

a general behaviour of the bowl from an elastic to elastoplastic or breaking stage will be observed. The values of these critical rotational speeds (*Table 8*) set up the fundamentals of the numerical analyses by the finite element method (FEM) developed to identify the critical stages and junctions. Mises equivalent stresses and normal hoop stresses reveal an increase with rotational speed more quickly for axial central load F_{a1} than F_{a2}. For the main part of the bowl named 'CI' the increase in the stress in the vicinity of the lower critical area 'a-c-r' is more intensive than for the main part of the bowl named 'CS' in the vicinity of the critical areas '1e–1z' (*Figs. 10–13*). At a rotational speed $n = 18000$ rpm an elastoplastic stage occurs in material behaviour in both the previous critical areas. At a rotational speed close to $n = 22000$ rpm the normal effective stresses exceed the breaking strength for large areas (*Figs. 11c, 12c, 13c*) and very probably the bowl breaks. In conclusion, for maximum technological load when the maximum rotational speed is $n = 8500$ rpm, this separator type offers a good reserve of load-carrying capacity.

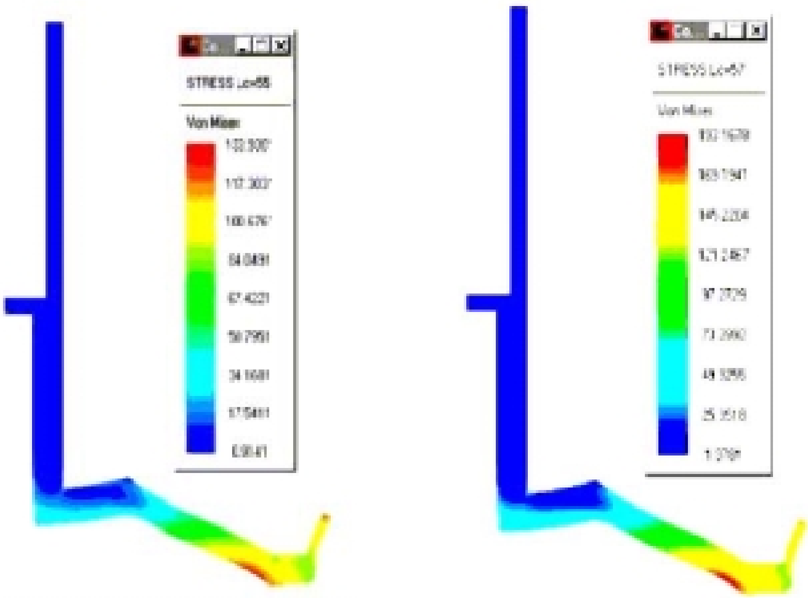

Fig. 11. Equivalent stress and deformed shape for the inner lower part of the bowl – FEM values at critical rotational speeds. a.) Case 55 (*Table 8*); b.) Case 57 (*Table 8*); c.) Case 59 (*Table 8*)

4. Conclusions

Excepting the deviations between the maximum analytical stresses and the FEM results, in the main junction (*Fig. 1*), a generally acceptable correlation was established between analytical, numerical and even experimental results (ANGHEL, IATAN, PASAT, 1998). The advantage of analytical analysis, like less CPU and memory space requirement, over the finite element method (FEM) is lessened by the overestimation of stress concentration state in critical junctions. The study reveals junctions corresponding to '9-8-7' respectively '3-4' (*Fig. 2b*) as areas under maximum stress beginning with the mounting of the bowl. A good reserve of load-carrying capacity and a safe operation make this type of separator bowl suitable for use under the mentioned technological conditions.

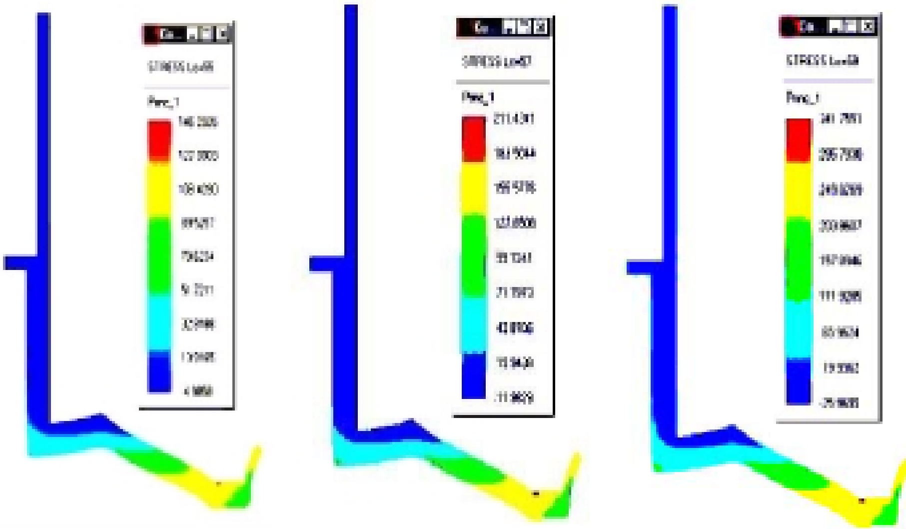

Fig. 12. Normal hoop stress and deformed shape for the inner lower part of the bowl – FEM values at critical rotational speeds. a.) Case 55 (*Table 8*); b.) Case 57 (*Table 8*); c.) Case 59 (*Table 8*)

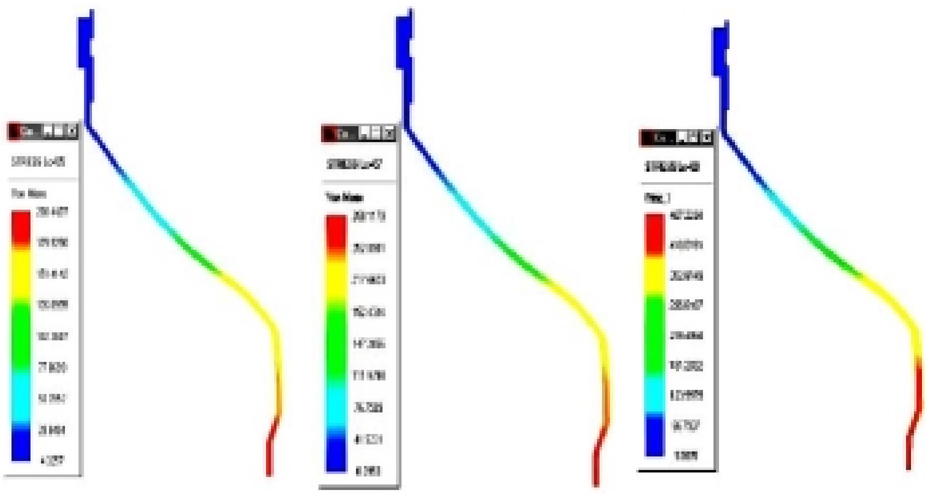

Fig. 13. Equivalent stress and deformed shape for the outside upper part of the bowl – FEM values at critical rotational speeds. a.) Case 55 (*Table 8*); b.) Case 57 (*Table 8*); c.) Case 59 (*Table 8*)

Table 7. Material properties

Parts of the bowl	Material*	$\sigma_{0.2}$ [MPa]	σ_r [MPa]	KCU [J/cm^2]	σ_a [MPa]
Outside 'CS'	OL 37.2k	230	360–400	27	150 . . . 160
Inner 'CI'	OL 37.2k	230	360–400	27	150 . . . 160

*Proceeding from STAS 500/2-80 and 2883/2/1-80 at ambient temperature $T = 20\ ^{\circ}\mathrm{C}$
$\sigma_a = \min\{\sigma_{0.2}/c_c;\ \sigma_r/c_r\}$ allowable stress at working temperature;
$c_c = 1.5$ safety factor for technical yield stress; $c_r = 2.4$ safety factor for breaking strength [3,4,5].

Table 8. Stresses and geometric positions at critical rotational speeds (*Fig. 10–13*)

Load case	Specific features	Conventional allowable stress	Maximum equivalent stresses	Geometric position
Case 55	Prestressed stage $F_{a1} = 1480$ N rotational speed $n = 15000$ rpm	$\sigma_a^* = 160$ [MPa]	146–202 [MPa]	'acr'on 'CS' and '1e–1z' on 'CI'
Case 57	Prestressed stage $F_{a1} = 1480$ N rotational speed $n = 18000$ rpm	$\sigma_{0.2}^* = 230$ [MPa]	193–290 [MPa]	'acr'on 'CS' and '1e–1z' on 'CI'
Case 59	Prestressed stage $F_{a1} = 1480$ N rotational speed $n = 22000$ rpm	$\sigma_r^* = 370$ [MPa]	311–463 [MPa]	'acr'on 'CS' and '1e–1z' on 'CI'
σ_a^* – allowable stress; $\sigma_{0.2}^*$ – technical yield stress; σ_r^* – breaking strength.				

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